When “International Consumption Correlations Puzzle” Meets “Kaldor’s Facts”: The Unbalanced Growth Approach

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Abstract

We explore the existence of “unbalanced” growth between the growth rate of consumption, that of capital owned and that of capital used, dynamically in open economy to explain the “international consumption correlation puzzle”, while the balanced growth is considered as the basic assumption for whole growth theory, with the support of empirical evidences such as “Kaldor Facts”. Keeping the traditional assumption of existence of “Kaldor Facts” in this paper with a specified two-good-and-two-country (developing-developed-countries) differential game model, we find that with both international good and capital flows which could be considered as a complete market, the growth rates of consumption for the developed and developing countries are not only different, but also diverged each other dynamically. This conclusion is generally held, regardless of development of financial market and/or existence of nontradable goods. These results are robust for both cases of capital flows and in more general models with endogenized rate of time preferences and endured physical capital.

Key Words: International Consumption Correlations Puzzle, Kaldor’s Facts, Unbalanced Growth, International Capital Movement, Growth Divergence

JEL Classification: O41, F43, F02, F20, O16, E20

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1. Introduction

“International consumption correlations puzzle” has become an important topic in recent decade. It says when the complete market model (perhaps with CRRA) predicts there exists the strong correlations in international consumption level and in growth rates of consumption per capita for different countries across time and states of nature. Current literature attempts to explain this puzzle through existence of non-tradable goods consumption, preference shocks, etc. These explanations, however, seem to have too many restrictions/assumptions which could reduce its generality for their explanations. We, therefore, propose to provide a more general explanation for such puzzle in this paper.

When one discusses about the growth rates of consumption, growth theory should be reviewed then. Although this “New” (endogenous) growth theory combines some development from international economics (Grossman and Helpman, 1989, 1990, 1992, etc.), some fundamental changes resulting from the dramatic change of international goods/factors flow are not revealed by the current growth study, which is the key issue in growth theories. Existence of such potential problems in growth theories could make some of important results questionable.

Moreover, many results from existing growth theories, including both neoclassical and endogenous growth approaches are based on the common and fundamental assumption of the internal balanced growth paths for individual countries with and without international factors flow. That means, they agree (or implicitly infer) that there exist the internal balanced growth rates between its growth rates of consumption, output and capital at least in the long-run for each country, whatever having international capital flow or not. This assumption was empirically supported by “Kaldor’s Facts” (1961), which indicate some empirical findings in the U.K. in 1960’s. They mainly consist of the following facts that (1) per capita output growth rate is roughly constant; (2) the capita-output ratio is roughly constant; (3) the real rate of return to capital is roughly constant; and (4) the shares of labor and capital in national income are roughly constant. While some of them are recognized as “stylized” facts (Jones, 2001), they are also ignored by some economists studying in “structure change” since they think they focus on the longer period than the facts apply. It causes us to think the suitability of such facts for our study in the commonly accepted period. Moreover, some empirical results are consistent with them, while some are not, even though within the U.S. (for example, Kongsamut, Rebelo and Xie, 2001).

As the matter of fact, dramatic increasing of international goods and capital flows in recent decades bring some significant change the allocation of goods and factors worldwide. For example, domestic consumption is fluctuating much more than before: resulting from increasing variety and quality of different goods produced in domestic and foreign countries, or from the international consumption externality trend (fashion), etc. International capital flow becomes larger and quicker. More importantly, the definition for capital has been distinguished clearly in our analysis, since the capital owned is not equal to the capital used in production. All those phenomena indicate the necessity to reexamine the existence of a stylized “balanced growth” for each country over reasonable period given greatly different environment (of international factors/goods flow) after forty years. That is, is the empirical support for the “balanced growth”, “Kaldor Facts”, still true? If yes, is there possible difference between the Kaldor’s definition and ours? If no, what is changed?

There are some researches concerning such “unbalanced” economic growth. Precisely, there are two approaches mainly on this topic. One is focus on internal “structure change” within a country and its effects (Baumol, 1967; Pasinetti, 1981; Park, 1995; Echevarria, 1997; Laitner, 2000; Kongsamut, Rebelo, and Xie, 2001; etc.). The other one is focus on development of a country, i.e. the process of catch-up for a developing country between different sectors (Hirschman, 1958; Murphy, Shleifer, and Vishny; 1989; Kriashna and Perez, 2005; etc.). Generally, both of them are focus on the “unbalance” between sectors, and greatly different from our discussion. We explore the “unbalanced growth” based on the definition used in all growth theory books: the path “where each variable of the model is growing at a constant rate” (e.g. Romer, 2001), and attempt to find such balanced growth is generally unreachable.

Obviously, it is not enough as well to analyze the impact of economic growth only in static text. General speaking, for the economic growth theory, it needs to discuss the dynamic context for economic growth in the complete open economy, i.e. in the economy with both international goods (trade) and international factor movement including international capital flow. This paper will attempt to do some research in this direction. That is, we will explore the dynamic effect of economic growth in the complete open economy. For simplicity, we assume that the world economy is a simple open economy. That is, there does not exist any restriction on goods and capital flow internationally, and but exist full restriction on international labor flow. Since it could be no significant difference to use neo-classical or endogenous growth models to find the effect of physical capital on economic growth, if we do not want to distinguish or discuss the role of knowledge (or human capital). Without loss of generality, we use the simple neo-classical growth model in this paper.

Generally, we think that the “balanced growth”, as a questionable assumption, could cause the negative effect widely to let many consequent results to be questionable, especially since it ignores the effect of international capital flow on economic growth which plays a basic and/or important role for economic growth in each country[1], whatever for the well-developed countries and developing countries. It is our main concern in this paper. We release this common assumption and attempt to find the dynamic effects of international capital flow on economic growth for each country and the world as a whole in the long-run.

When we focus on the effect of capital on economic growth, the impact of consumption on economic growth could not be ignored. Due to the role of saving rates in growth, we should see whether exogenity or endogenity of rates of time preferences has significant effects on economic growth or not. We will, therefore, check the robustness of

[1] Although Solow (1958) shows the less important role of physical capital on growth, comparing that of technology, empirically, the importance of international capital flow seems underestimated for long time. Our paper will show such importance.

results from exogenously given rates of time preferences under the situation with endogenous rates of time preferences.

We reveal that the general existence of the “unbalanced growth paths” between the consumption and the capital owned by each country over time dynamically. Moreover, given a developing-developed-countries framework, we find that the developed country obtains the significant benefit in the form of the higher growth rate of capital owned and lower consumption growth rate, comparing with the developing country in the two-country world. However, the developing country also obtains a higher growth rate than that in autarky, as the incentive for the developing country to open its door. Moreover, the existence of conditional divergence of growth levels between the developing country and the developed country has been derived.

Our paper attempts to solve such fundamental problem in the following way. Section 2 shows basic model and assumptions in this paper. Section 3 finds dynamic Nash equilibrium results under open economy with one-way capital flow, comparing those under autarky. Section 4 will give the corresponding results in two-way capital flows at the Nash equilibrium. The comparison for these results is done in this section. Then, Section 5 can examine the above results in the situation with endogenous rates of time preferences. Section 6 will extend our results to discuss the famous question in the long time: the convergence of growth rates in different countries in the long-run. Since many debates on this question are based on the common assumption of holding the internal balanced growth rates, our results should be interesting. We conclude our findings and give some suggestions for the future research in Section 7.

2. Basic Framework and Basic Assumptions

2.1. Basic Framework

The significant advantage of neo-classical model of growth is simple and useful. The advantage of new endogenous model is to create intermediate (capital) goods or capital goods index or other parameters showing the technology and to describe the technological progress. In some sense, the new endogenous growth theory focuses on how to describe and explain the effect of technological progress on economic growth. Therefore, if we only need to find the impact of capital movement on economic growth, the relative complicate model of new endogenous growth models could be replaced by the neo-classical model of growth to indicate the key of problems we are interesting. In this paper, we just use the neo-classical growth model to inquire my problems based on the above reasons. Precisely, we use a two-good-and-two-country neo-classical model. In this framework, we still assume the existence of the "Kaldor Facts", while we show our suspicion of such facts over reasonable period empirically in the other paper.

2.2 Basic assumptions

Here, we try to build a basic model and give its competitive equilibrium results, which is viewed as

the basic parts of growth theory research. We adopt such basic assumptions used in the other endogenous economic growth literatures. For simplicity, we assume that (1) each country has one good which is not overlapped by the other country's in this two-country world; and (2), as we mentioned before, only potential goods and capital mobility between two countries. That is, there is no labor or human capital migration between countries. To let the problem simpler and clearer, we assume further that (3) there are a developed country 1, with extra capital, with negative net export (trade deficit)[2], and

[2] This assumption will result in the negative foreign reserve or debt in the government account. However, for simplicity, we keep this simple assumption in this paper. Same as for the developing country does.

relative lower capital return[3], and a developing country 2, with capital shortage, with positive net export (trade surplus), and relative higher capital return. As we discussed above, we do not attempt to formulate the technology, and rather, let it be a fixed parameter for each country. In this way, we simplify the difference of productivity between both countries as the difference of the fixed parameters between two countries' production functions. It means the technology has been not changed over time (at least over our research period). This assumption seems strong, but it does not affect our focus in this paper. Moreover, we keep this assumption as same as many research papers in order to compare our results.

We suppose that the depreciation rate of capital is zero for simplicity. Normally, the labor is assumed the fixed parameter or the parameter with known initial level and fixed growth rate in the growth theory. Therefore, it is a known parameter. In this paper, we assume the labor in both countries is unity.

According to Charles Jones's paper (1999), the shape of production function or rate does matter for our analysis. Thus, we use the basic assumptions for the production function which are commonly used in the growth theory: (1) convexity of production, and (2) constant return to scale, i.e. not increasing returns, without loss of generality. Precisely, we use the simple Cobb-Douglas production function as the form of production functions for both countries. That is, $Y_i = A(or\ B)L_i^{\beta} K_i^{\alpha}$, which A (or B)>0 is used as the technological parameter for each country, L is the labor used and K is the capital used, i is refer to each country and same thereafter. As our assumption above, the production function is assumed as homogenous degree of one. The utility function is assumed as the following: $U(C_i) = \ln C_i$, which C is the consumption in each country. And the individual in each country is assumed to maximize her utility as $Max \int_0^{\infty} U(C_i) e^{-\rho_i t} dt$, which ρ is the fixed rate of time preference, and it is not necessary same for each country.

It is very important in this paper that we divide the capital into the capital owned (by each country or firm) and the capital used in the production. Therefore these two kinds of

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[3] This kind of productivity is expressed as a single parameter, A or B, which could result from all of possible reasons. My assumption for it is to try to explain the international investment and/or capital flow, which should be supported by the relative marginal return of inputs, and here, the relative productivity.

capital should be changed in different rates, which these are indifferent in autarky. Following the "Kaldor facts", the capital having same growth rate as those of consumption's and output's at the steady state should be the capital used in the production function.

3. Dynamic Nash Equilibrium with One-way Capital Flow

Now, we show the one-way capital flow at dynamic Nash equilibrium first in order to compare the following Nash equilibrium with two-way capital flow. In both equilibra, we use the above assumption for the fixed rates of time preferences first. We will check our results with varied rates of time preferences after. To deepen our understanding for the following comparison, we need to have the results for autarky with the same constant rate of time preferences as the reference.

3.1 Equilibrium in Autarky:

Country 1: Production function: $Y_1 = AL^{\beta}{}_1 K_1^{\alpha}$,

Here, let the wealth W is equal to the physical capital K in Country 1. That is, $W_1(t) = K_1(t)$, and the utility function as: $U(C_i) = \ln C_i$. Let the $L_1 = L_2 = 1$ in the both countries as one of the basic assumptions, and A will be the relative technology index between two countries, then we have:

$$Max \int_0^{\infty} U(C_1) e^{-\rho_1 t} dt = \int_0^{\infty} \ln C_1 e^{-\rho_1 t} dt$$

subject to $\dot{K}_1(t) = Y_1(t) - C_1(t) = AK_1{}^{\alpha}(t) - C_1(t)$

Therefore, we have the current-value Hamilitonian function as followings:

$$H = \ln C_1 + \lambda [AK_1^{\alpha} - C_1]$$

we have:

$$\frac{\dot{C}_1}{C_1} = -\rho_1 + A\alpha K_1^{\alpha-1}, \qquad \text{where } r_1 = A\alpha K_1^{\alpha-1} = \text{MPK for Country 1}$$

Therefore, the balanced growth rate for Country 1 is:

$g_1 = \frac{\dot{C}_1}{C_1} = \frac{\dot{K}_1}{K_1} = (r_1 - \rho_1)$. Then $\frac{C_1}{K_1} = \frac{1-\alpha}{\alpha} r_1 + \rho_1$, since $\frac{\dot{C}_1}{C_1} = \frac{\dot{K}_1}{K_1}$

For the Country 2, the production function: $Y_2 = L^{\beta}{}_2 K_2^{\alpha}$,

we have:

$$Max \int_0^{\infty} U(C_2) e^{-\rho_2 t} dt$$

s.t. $\dot{K}_2(t) = Y_2(t) - C_2(t) = K_2{}^{\alpha}(t) - C_2(t)$

Similarly, we have:

$\frac{\dot{C}_2}{C_2} = -\rho_2 + \alpha K_2^{\alpha-1}$, where $r_2 = \alpha K_2^{\alpha-1}$ = MPK for Country 2

Therefore, the balanced growth rate for Country 2 is:

$g_2 = \frac{\dot{C}_2}{C_2} = \frac{\dot{K}_2}{K_2} = (r_2 - \rho_2)$. Then $\frac{C_2}{K_2} = \frac{1-\alpha}{\alpha} r_2 + \rho_2$, since $\frac{\dot{C}_2}{C_2} = \frac{\dot{K}_2}{K_2}$.

Remarks: From above we know when the rates of time preference for the both countries are same; there exists the same growth rate, with the same rate of return of physical capital. If the rates of time preference are not same, however, the growth rates for two countries are not same even if the interest rate r is same for the both countries. Since there is not any international goods trade and capital movement (i.e. in autarky), such growth rate can be kept stable and the balanced growth rates for each country can be expected.

3.2 Define r, w, p as r=r(K), w=w(K), and p=p(K)

Now we can see the competitive equilibrium under general situation: international goods trade and international capital flow are introduced. Since the international capital flow is added into this two-country world, we need to find the amount of international capital flow first, before we try to find the growth rate for the both countries.

As we assume before that there are two countries: Country 1, with initial lower capital returns and as a net capital outflow country, and, Country 2, with initial higher capital returns and as a net capital inflow country. There is no restriction on trade balance. Then the individual production function will be:

Country 1: $F_1 = Y_1 = AL^{\beta}{}_1 K_1^{\alpha}$; Country 2: $F_2 = Y_2 = BL_2^{\beta} K_2^{\alpha}$,

where: $K_1 = K_{11}^{\gamma} K_{21}^{1-\gamma}$, $\mathrm{K}_2 = K_{22}^{\theta} K_{12}^{1-\theta}$, and K_{11} , K_{12} are the capital good owned by Country 1 and used in Country 1 and 2, respectively; similarly, K_{22} , and K_{21} are the capital good owned by Country 2 and used in Country 1 and 2, respectively. Then, there are heterogeneous capital goods K_1 and K_2 aas different intermediate goods used in different countries/production. Supposed that Country 1 and 2 have the same α and β.

According to the definitions above, we can define two kinds of new definitions for consumption goods and capital goods. For capital goods, except for K_1 and K_2 defined as above as capital used in each country, we still have K_a and K_b as the capital owned by Country 1 and 2 respectively. That is, $K_a = K_{11} + K_{12}$, $K_b = K_{22} + K_{21}$. Certainly, we can have the similar definitions for the consumption goods as well in the following sections.

Before we attempt to find the growth rates, we need to find the prices[4] for consumption and capital goods first. For this purpose, we set the following assumptions : Let: the price for the output for Country 1 is: p_1, the corresponding price for Country 2 is unity; the price of capital owned by Country 1 is: r_1 , the corresponding is unity.
Then the manufacturer in Country 1 will maximize their profit:

$$\underset{K_{11}}{\mathrm{Max}}\ \mathrm{p}AK_1^{\alpha} - rK_{11} - p_2 K_{21} = \mathrm{p}AK_{11}^{\alpha\gamma} K_{21}^{(1-\gamma)\alpha} - rK_{11} - K_{21}$$

Similarly, for Country 2:

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[4] Here, we assume the price for consumption goods is same as that for the output for the same country. Moreover, we do not want to distinguish the price and rent (or interest, or investment return) for the capital goods, since the price of capital can be considered as the capital return, which can be showed as any form. For simplicity, we just use the "price" to express them for both countries.

$$\underset{K_{22}}{\text{Max}} \quad BK_2^{\alpha} - rK_{12} - K_{22} = BK_{22}^{\alpha\theta} K_{12}^{(1-\theta)\alpha} - rK_{12} - K_{22}$$

Now we have the following three conditions:

For the same kind of capital, it should have the same rate of return in the different country, since there is a perfect international capital flow. Each marginal product for capital (MPK) equals the return of that capital. There exist, however, two different goods and their MPK will be expressed in the different goods. Therefore we use MVPK instead of MPK for each good (and country). Then we can write as follows:

(1) $$MVP_{K_{11}} = MVP_{K_{12}}$$

(2) $$MVP_{K_{21}} = MVP_{K_{22}}$$

From (1) and (2), we have:

(3) $$\frac{K_{11}}{K_{21}} = (\tfrac{1}{\Lambda})\frac{K_{12}}{K_{22}} = Z, \qquad \text{where}: \Lambda = \frac{(1\text{-}\gamma)(1\text{-}\theta)}{\gamma\theta}$$

then we have,

(4) $$K_{11} = ZK_{21},$$

(5) $$K_{12} = \Lambda ZK_{22}$$

And assume the total capital in the world is known, we have the equivalence of capital as the third condition we mentioned:

(6) $$K = K_a + K_b = (K_{11} + K_{21}) + (K_{22} + K_{12})$$

Therefore, we know: r, K_{12}, K_{22} are the function of Z, and K_{11}, K_{21} are the function of Z and p. So the total capital K is the function of Z and p.

For the given situation we described before, that Country 2, as a developing country, did not have capital outflow to the developed country, Country 1. That is, K_{21}=0. Then we can modify the equations (1)-(6) and (A1)-(A12) in Appendix A, while Equation (2) disappears.

Similarly, we can find the wage as the function of whole capital.

$$w_1L_1 = q_1[F_1(K_1, L_1) - (\frac{\partial F_1}{\partial K_1})K_1]$$

Given our assumptions as above, we have

(7) $$w_1 = pA(1-\alpha)K_1{}^{\alpha} = pA(1-\alpha)K_{11}^{\alpha\gamma}K_{21}^{\alpha(1-\gamma)} = w_1(K,Z)$$

Similarly, we have the corresponding results for Country 2:

(8) $$w_2 = F_2 - (\frac{\partial F_2}{\partial K_2})K_2 = B(1-\alpha)K_2{}^{\alpha} = B(1-\alpha)K_{22}^{\alpha\theta}K_{12}^{\alpha(1-\theta)} = w_2(K,p,Z)$$

Now we can attempt to find the growth rates for the both countries.

3.3 Equilibrium in Open Economy with the Exogenous Rate of Time Preferences

Now, we define the consumption in both countries in the following way.

Consumption: in Country 1, the representative resident consumes both good 1 (produced at home) and good 2 (imported from oversea). The value of her consumption, in terms of good 1, is:

$q_1C_{11} + C_{21}$, where: C_{11} is the good Country 1 produced and comsumed in Country 1,
C_{21} is the good Country 2 produced and comsumed in Country 1.

Consumption: in Country 2. The value of her consumption, in terms of good 1, is:

$q_1C_{12} + C_{22}$, where: C_{12} is the good Country 1 produced and comsumed in Country 2,
C_{22} is the good Country 2 produced and comsumed in Country 2.

Given the utility function is simplified as: U (C_{ii}, C_{ji}) = lnC_{ii} +lnC_{ji}, thus, we can find the both country's (or their representative's) utilities are maximized as follows.

Country 1: (for each individual in Country 1)

Max $\int_0^{\infty}(\ln C_{11} + \ln C_{21})e^{-\rho t}dt$

Subject to $\dot{K}_a = rK_a + w_1(K)L_1 - pC_{11} - C_{21}$, where: $K_a = K_{11} + K_{12}$

For Country 2, we have the corresponding analysis, thus:

Max $\int_0^{\infty}(\ln C_{22} + \ln C_{12})e^{-\rho t}dt$

Subject to $\dot{K}_b = K_b + w_2(K)L_2 - C_{22} - pC_{12}$, where: $K_b = K_{22} + K_{21}$

Then we can see that there exist two kinds of growth rates of consumption for each country. That is, the growth rate of consumption for one goods (produced in one country and used in both countries):

$\frac{\dot{C}_a}{C_a} = \frac{(C_{11}+C_{12})^{\dot{}}}{(C_{11}+C_{12})} = \frac{\dot{C}_{11}}{(C_{11}+C_{12})} + \frac{\dot{C}_{12}}{(C_{11}+C_{12})}$ for Country 1; and

$\frac{\dot{C}_b}{C_b} = \frac{(C_{22}+C_{21})^{\dot{}}}{(C_{22}+C_{21})} = \frac{\dot{C}_{22}}{(C_{22}+C_{21})} + \frac{\dot{C}_{21}}{(C_{22}+C_{21})}$ for Country 2; and the growth rate of consumption of each country: $\frac{\dot{C}_1}{C_1} = \frac{(C_{11}+C_{21})^{\dot{}}}{(C_{11}+C_{21})} = \frac{\dot{C}_{11}}{(C_{11}+C_{21})} + \frac{\dot{C}_{21}}{(C_{11}+C_{21})}$ for Country 1, and

$\frac{\dot{C}_2}{C_2} = \frac{(C_{22}+C_{12})^{\dot{}}}{(C_{22}+C_{12})} = \frac{\dot{C}_{22}}{(C_{22}+C_{12})} + \frac{\dot{C}_{12}}{(C_{22}+C_{12})}$ for Country 2. Similarly, we have the similar definitions for growth rates of capital owned (by one country), and used (by one country):

$\frac{\dot{K}_a}{K_a} = \frac{(K_{11}+K_{12})^{\dot{}}}{(K_{11}+K_{12})}$, and $\frac{\dot{\mathrm{K}}_{\mathrm{b}}}{\mathrm{K}_{\mathrm{b}}} = \frac{(K_{22}+K_{21})^{\dot{}}}{(K_{22}+K_{21})}$; and

$\frac{\dot{K}_1}{K_1} = \frac{(K_{11}^{\gamma}K_{21}^{1-\gamma})^{\dot{}}}{(K_{11}^{\gamma}K_{21}^{1-\gamma})}$, and $\frac{\dot{\mathrm{K}}_2}{\mathrm{K}_2} = \frac{(K_{22}^{\theta}K_{12}^{1-\theta})^{\dot{}}}{(K_{22}^{\theta}K_{12}^{1-\theta})}$. From these distinctions, we can find the growth rates precisely. According to our calculation, we have:

for Country 1: $\frac{\dot{C}_1}{C_1} = (\frac{2}{q_1+1})(p_1-\rho)$, $\frac{\dot{C}_a}{C_a} = (\frac{p_1+1}{2}) - \rho$;

for Country 2: $\frac{\dot{C}_2}{C_2} = (\frac{2}{q_1+1})(1-\rho)$, $\frac{\dot{C}_b}{C_b} = (\frac{p_1+1}{2q_1}) - (\frac{1}{q_1})\rho$.

Now, as we mentioned in the beginning of Section 2.2 "Basic Assumptions" that Country 1 is a developed country, and Country 2 is a developing country, and then we assume here that the capital flow from Country 2 to Country 1 is zero, without of generality. That is, $K_{21} = 0$. To make the situation of international capital movement reasonable, we assume that the capital return in Country 2 is higher than that in Country 1, so A<B given the same form of production function for two good (i.e. in two countries). Another related assumption is that $q_1>1$, supposing that the developed country can give more advanced goods or more expensive goods, and that the developed country has higher wage rate which pulls its product's price.

Then we have the following proposition and lemmas.

Proposition 1: *In the open economy with international goods and capital flows, with above assumptions, there exists <u>unbalanced growth rates (paths) for each country</u> between its consumption and its own capital.*

Moreover, there are the following Lemmas.

Lemma 1. *Growth rates of capital used in individual country are kept same as that in autarky, and equal to that of the whole world.*

Lemma 2. *The growth rates of each country's own capital in the open economy could be higher than that at autarky, while Country 1, the developed country, will obtains the higher rate.*

Lemma 3. *The growth rates of consumption with international goods and capital flows are no more than those in autarky.*

Remarks: The results in Proposition 1 and Lemma 1-3 above tell us that in the open economy the growth rates for the individual country could be underestimated for long time under the assumption of "balanced growth rates (paths)". Moreover, there shows the incentive for the developing country to attracts the foreign investment (e.g. FDI) to boost its economic growth.

Furthermore, from our results, such unbalanced growth should be general situation over its economic growth, at least in short run. This result is surprising and interesting. The above lemmas show that if the domestic consumption in Country 2, the developing country, is not less than its export (it is a reasonable assumption), the growth rate capital owned by Country 1's investors will be higher than that for Country 2's. It could be the important incentive for Country 1's investors to invest in Country 2, ignoring the difference in their population. For Country 2, since it wants to catch up the developed countries, it needs more capital and has to accept this fact. It could leave us another question: whether the capital owned by Country 2 will not reach the amount of capital owned by Country 1 and will need the Country 1's capital forever, and whether developing countries, e.g. Country 2, can never catch up the developed country, e.g. Country 1, in the general sense. Answering such questions is not the task for this paper, but such questions are very interesting.

4. Dynamic Nash Equilibrium with Two-way Capital Flow

4.1 Define International Capital Flow and Investment Return

We continue to discuss the situation for the dynamic Nash equilibrium under different situations: two-way capital flow. Since the former Nash Equilibrium consider the reaction from the trading partner which is more reasonable in the two-country world. Now we have something different with the above cases. Each country’s owned capital change, not the capital itself, is the function of the other country’s owned capital. Also due to this feather, we can find the international capital flow at equilibrium. Then we can try to discuss the following two situations for the corresponding results.

Now, we try to find the international flowing capital as the function of each country’s owned capital. We know the following facts used before, assume the definitions for different capital, consumption and labor are same as defined in the previous sections.

For Country 1: $Y_1 = AL^{\beta}{}_1 K_1^{\alpha}$, where $K_1 = K_a - K_{12}$,

i.e. the capital used in Country 1 = the capital owned by Country 1's investors - the capital owned by Country 2's investors.

For Country 2: $Y_2 = BL_2^{\beta} K_2^{\alpha}$, where $K_2 = K_b + K_{12}$

There are the exactly same good and are supposed that Country 1 and Country 2 have the same α and β. There are two state variables: K_a, K_b and two control variables: C_{11}, C_{22}, so they have

$\dot{K}_a = PY_1 - PC_{11} + PC_{12} - C_{21} + rK_{12}$, $\dot{K}_b = Y_2 - C_2 + C_{21} - PC_{12} - rK_{12}$.

Let's consider the national income like GNP or wealth, such as:

GNP in Country 1 = output in Country 1 + return from investment in Country 2.

These two formula are expressed before. However, since in the competitive equilibrium, each player in the market never responds to the other's behavior. Therefore we use other formula representing same idea in our problem. Here is the case we can use such formula directly.

Since as before, the whole world economy, the both MVPKs must be equal:

$$B\alpha(K_b + K_{12})^{\alpha-1} = PA\alpha(K_a - K_{12})^{\alpha-1} = r(t)$$

then:

$$K_{12} = \frac{K_a - EK_b}{1+E}, \quad \text{where } E = \left(\tfrac{B}{PA}\right)^{\frac{1}{\alpha-1}}, \text{ therefore } Z = \frac{E}{1+E}$$

Therefore, we can see such facts: $K_{12} = f(K_a, K_b, P)$, $E = f(P)$, $Z = f(P)$. Moreover, from the above results we obtain:

$$\dot{K}_a = \frac{PAE^{\alpha}}{(1+E)^{\alpha}}(K_a + K_b)^{\alpha} + r\left(\frac{K_a - EK_b}{1+E}\right) - PC_{11} + PC_{12} - C_{21}$$

$$\dot{K}_b = \frac{B}{(1+E)^{\alpha}}(K_a + K_b)^{\alpha} - r\left(\frac{K_a - EK_b}{1+E}\right) - C_{22} + C_{21} - PC_{12}$$

From above, we can see that since $\left(\frac{B}{PA}\right) = \left(\frac{K_1}{K_2}\right)^{\frac{1}{\alpha-1}} = E$, and we assume (a) $K_a > K_b$, (b) $K_1 > K_2$, (3) $K_{12} > 0$ 0<α<1, then E>1, and assume, therefore as we supposed. So, PA > B.

Then, we substitute K_{12} into $r(t)$, we get the $r(t)$, and substitute it into the growth rate we got before:

$$r(t) = \frac{B\alpha}{(1+E)^{\alpha-1}}(K_a + K_b)^{\alpha-1} = PA\alpha(\frac{E}{1+E})^{\alpha-1}(K_a + K_b)^{\alpha-1}$$

4.2 Equilibrium with the Exogenous rate of Time Preferences:

<u>Country 1:</u>

Max $\int_0^\infty (\ln C_{11} + \ln C_{21})e^{-\rho t} dt$

Subject to $\dot{K}_a = \frac{PAE^\alpha}{(1+E)^\alpha}(K_a + K_b)^\alpha + r(\frac{K_a - EK_b}{1+E}) - PC_{11} + PC_{12} - C_{21}$

<u>Country 2:</u>

Max $\int_0^\infty (\ln C_{22} + \ln C_{12})e^{-\rho t} dt$

Subject to $\dot{K}_b = \frac{B}{(1+E)^\alpha}(K_a + K_b)^\alpha - r(\frac{K_a - EK_b}{1+E}) - C_{22} + C_{21} - PC_{12}$

Then we have the following proposition and lemmas.

Proposition 2: *In the Nash equilibria with free goods and capital flows, given all above assumptions, it exists <u>unbalanced growth rates (paths) for each country</u> between its consumption and its own capital.*

Moreover, there are the following Lemmas.

Lemma 4. *The growth rates of capital used in individual countries are close to each other, if not same, and are higher than those in autarky. They are also close to, if not equal to, that of the whole world.*

Lemma 5. *The growth rates of each country's own capital are different from those in autarky. Precisely, the growth rate of Country 1's own capital in open economy could be higher than in autarky, while that of Country 2's own capital is less than that of in autarky.*

Lemma 6. *The growth rates of consumption with international goods and capital flows are less than those in autarky.*

Proof: see Appendix 2.

Remarks: The results in Proposition 2 and Lemma 4-6 tell us that in open economy the growth rates for individual countries could be underestimated with holding the assumption of the balanced growth paths.

From the results of Section 3 and 4, we can see that in different scenarios, we can get the similar results that there exists the unbalanced growth rates when we use the capital in different ways. Even due to our assumptions, such unbalanced growth rates can not be considered to be true universally, we can find the existence widely of such fact, especially when we recognize the situations we mentioned in this paper do commonly exist. Although the definition for the used capital in each country is different in Section 3 and 4, which causes the difficulty to compare their results, we still can find the following interesting facts.

5. Equilibria with Endogenous Rates of Time Preferences

Now we add two new assumptions for the both countries' situation as follows:

(1) The two countries has two different utility functions as before:

for Country 1: $U(C_{11}, C_{21}) = \ln C_{11} + \ln C_{21}$;

for Country 2 : $U(C_{22}, C_{12}) = \ln C_{22} + \beta \ln C_{12}$, where β>0

(2) The rates of time preference for the two countries varies with time:

for Country 1: $\dot{\Delta}_1 = \delta_1[U(C_{11}, C_{21})] = a_1 + b_1 U(C_{11}, C_{21}) = a_1 + b_1(\ln C_{11} + \ln C_{21})$,

for Country 2 :

$$\dot{\Delta}_2 = \delta_2[U(C_{22}, C_{12})] = a_2 + b_2 U(C_{22}, C_{12}) = a_2 + b_2(\ln C_{22} + \beta \ln C_{12}),$$

where all a, b, β>0.

5.1 One-way Capital Flow Nash Equilibrium with Endogenous Rates of Time Preferences

Here we adopt all assumptions for Section 2, i.e. for the competitive equilibrium with the exogenous determined rate of time preference. It is worth to note that the definition for the capital used in an individual country in Section 2 is different with that in Section 3. But this difference does not change our conclusion.

Then, we adapt the problem above to the followings:

<u>Country 1:</u>

Max $\int_0^\infty U(C_{11}, C_{21}) e^{-\Delta_1(t)} dt$

Subject to $\dot{K}_a = rK_a + w_1(K)L_1 - pC_{11} - C_{21}$

$$\dot{\Delta}_1 = \delta_1[U(C_{11}, C_{21})] = a_1 + b_1 U(C_{11}, C_{21}) = a_1 + b_1(\ln C_{11} + \ln C_{21})$$

Then we get the following basic results:

$$\frac{\dot{C}_1}{C_1} = r(K) - \frac{a_1 e^{-\Delta}}{PC_1\lambda_1} - \frac{\dot{P}}{P} = r(K) - \frac{a_1}{1 + b_1 e^{\Delta}\lambda_2} - \frac{\dot{P}}{P}$$

$$\frac{\dot{K}_a}{K_a} = r + \frac{w_1(K) - pC_{11} - C_{21}}{K_a}$$

It shows that the growth rate of consumption will depend on the total capital K, and the time t, since λ_1, and λ_2 are the function of time.

Remarks: This result **does,** however, **depend on the form of the endogenous rate of time preference:** δ. Since the existence of it depends on the function form (if assume a=0, it will disappear in the expression of the growth rate of consumption). As we can see, when we let :

$$\dot{\Delta}_1 = \delta_1[U(C(t))] = b_1 U_1 = b_1(\ln C_{11} + \ln C_{21}) \neq a_1 + b_1 U_1 = a_1 + b_1(\ln C_{11} + \ln C_{21})$$

Then, $\frac{\dot{C}_1}{C_1} = r - \frac{\dot{P}}{P}$, which is larger than that before, while the growth rate of capital is affected.

Let us see the situation for Country 2:

Country 2 :

Max $\int_0^\infty (\ln C_{22} + \beta \ln C_{12}) e^{-\Delta_2(t)} dt$

Subject to $\dot{K}_b = K_b + w_2(K)L_2 - C_{22} - pC_{12}$

$$\dot{\Delta}_2 = \delta_2[U(C_{22}, C_{12})] = a_2 + b_2 U(C_{22}, C_{12}) = a_2 + b_2(\ln C_{22} + \beta \ln C_{12})$$

thus,

$$\frac{\dot{C}_2}{C_2} = r - (\frac{a_2 e^{-\Delta}}{\lambda_3 C_2}) = r - (\frac{a_2}{1 + b_2 e^{\Delta} \lambda_4})$$

$$\frac{\dot{K}_b}{K_b} = 1 + \frac{w_2(K) - C_{22} - pC_{12}}{K_b}$$

Similarly, this result **does** also **depend on the form of the endogenous rate of time preference:**δ. Since if we define the function form of $\dot{\Delta}_2$ as the following way :

$$\dot{\Delta}_2 = \delta_2[U(C(t))] = b_2 U_2 = b_2(\ln C_{22} + \ln C_{12}) \neq a_2 + b_2 U_2 = a_2 + b_2(\ln C_{22} + \ln C_{12}),$$

we can see $\frac{\dot{C}_2}{C_2} = r$, which is larger than that before, while the growth rate of capital is affected, as the situation of Country 1 above.

From the above we can derive the following proposition and lemma.

Proposition 3. *In the open economy with international goods and capital flow, with endogenized rates of time preference assumed and all above assumptions, there still exists unbalanced growth paths for each country (and probably for the world) between their consumption and their own capital.*

Similarly as before, we also have the following lemma.

Lemma 7. The growth rate of consumption for each country is always different, whatever at the steady state ($\dot{P}_x = 0$) or not. The same situation applies on the growth rates of capital owned by each country.
Proof. See **Appendix 3**.

Now we can compare these results with those in the open-loop Nash equilibrium in Section 3, since we have exactly same assumptions and formulation. We can see that our results for the unbalanced growth are robust.

5.2 Two-way Capital Flow Nash Equilibrium with Endogenous Rates of Time Preferences

Now we hold the same assumptions as before in Section 3 for the dynamic equilibrium for the both countries. Please note the definition for the capital used in an individual country in Section 3 is different with that in Section 2, even this change does not affect our conclusions generally.

Then, we change the problem above to the followings:

Country 1:

Max $\int_0^\infty U(C_{11}, C_{21}) e^{-\Delta_1(t)} dt$

Subject to $\dot{K}_a = \frac{PAE^\alpha}{(1+E)^\alpha}(K_a + K_b)^\alpha + r(\frac{K_a - EK_b}{1+E}) - PC_{11} + PC_{12} - C_{21}$

$$\dot{\Delta}_1 = \delta_1[U(C_{11}, C_{21})] = a_1 + b_1 U(C_{11}, C_{21}) = a_1 + b_1(\ln C_{11} + \ln C_{21})$$

Then $\frac{\dot{K}_a}{K_a} + \frac{\dot{P}}{P} + \rho > \frac{\dot{C}_1}{C_1}$.

There still exists the problem of definition of **endogenous rate of time preference:δ**, as we discussed in Section 5.1. We can see that if we change the formulate of δ as the following way:

$\dot{\Delta}_1 = \delta_1[U(C(t))] = b_1 U_1 = b_1(\ln C_{11} + \ln C_{21}) \neq a_1 + b_1 U_1 = a_1 + b_1(\ln C_{11} + \ln C_{21})$,

we will have: $\frac{\dot{K}_a}{K_a} + \frac{\dot{P}}{P} = \frac{\dot{C}_1}{C_1}$. Therefore, this new result is exactly same as that in the case above using constant rate of time preference, although it is the case of varying rate of time preference. But until now, this conclusion for the same result between the both cases for the rates of time preference only holds for the growth rate of consumption, not for the growth rates of different capitals since we do not check those rates.

For Country 2, we have:

Max $\int_0^\infty (\ln C_{22} + \beta \ln C_{12}) e^{-\Delta_2(t)} dt$

Subject to $\dot{K}_b = \frac{B}{(1+E)^\alpha}(K_a + K_b)^\alpha - r(\frac{K_a - EK_b}{1+E}) - C_{22} + C_{21} - PC_{12}$

$$\dot{\Delta}_2 = \delta_2[U(C_{22}, C_{12})] = a_2 + b_2 U(C_{22}, C_{12}) = a_2 + b_2(\ln C_{22} + \beta \ln C_{12})$$

Then $\frac{\dot{K}_b}{K_b} + \rho > \frac{\dot{C}_2}{C_2}$.

The problem of definition of **endogenous rate of time preference:δ**, has the exactly same effect as above. We will see that if we change the formulate of δ as the following way :

$\dot{\Delta}_2 = \delta_2[U(C(t))] = b_2 U_2 = b_2(\ln C_{22} + \ln C_{12}) \neq a_2 + b_2 U_2 = a_2 + b_2(\ln C_{22} + \ln C_{12})$

thus the conclusion will be changed as : $\frac{\dot{K}_b}{K_b} + \rho \equiv \frac{\dot{C}_2}{C_2}$. Therefore, this new result is exactly same as that in the case above using constant rate of time preference, although it is the case of varying rate of time preference. But until now, similarly as we mentioned before, this conclusion for the same result between the both cases for the rates of time preference only holds for the growth rate of consumption, not for the growth rates of different capitals since we do not check those rates.

Proposition 4. *In the open economy with international goods and capital flow, when varied rates of time preference assumed, given all above assumptions, there still* <u>*exists unbalanced growth rates (paths) for each country (and probably for the world)*</u> *between their consumption and their own capital.*

Similarly as before, we also have the following lemma.

Lemma 8. *The growth rates of consumption for each country are always different, whatever at the steady state (* $\dot{P}_x = 0$ *) or not. The same situation applies on the growth rates of capital owned by each country.*

Now we can compare these results with those in the Nash equilibrium in Section 4, with complete same assumptions and formulation. We can find that although we can not get explicit expressions for the growth rates for consumption and capital owned by each country, the general result we obtained before is still held: there exists the "unbalanced growth" widely.

6. Our Results for Convergence of Growth Rates between Different Countries

From our results in Section 3 and 4, we can see the dynamic trend of these growth rates. In the open-loop Nash equilibrium, we can find the following results.

Lemma 9. The growth rates of consumption are different among each country and the world. Particularly, the growth rate of consumption of Country 1 is always less than that of Country 2, whatever at the steady state or not, given the commodities have the different prices, i.e. $p_1 \neq 1$.

Lemma 10. The growth rates of each country's own capital will depend on the proportion of its own consumption and export. Precisely, let the growth rates of capital owned by Country 1 and Country 2 is g_1 and g_2, respectively, we have $g_1 > g_2$, if $C_{22} \geq C_{21}$; and vice versa.

For the situation of closed-loop Nash equilibrium in Section 4, we have the following results:

Lemma 11. The growth rates of consumption are different among each country and the world. Particularly, the growth rate of consumption of Country 1 is always less than that of Country 2, whatever at the steady state or not.

Now, the change of the capital owned by each country provides the following dynamic system:

$$\dot{K}_a = \frac{PAE^\alpha}{(1+E)^\alpha}(K_a + K_b)^\alpha + r(\frac{K_a - EK_b}{1+E}) - PC_{11} + PC_{12} - C_{21}$$

$$\dot{K}_b = \frac{B}{(1+E)^\alpha}(K_a + K_b)^\alpha - r(\frac{K_a - EK_b}{1+E}) - C_{22} + C_{21} - PC_{12}$$

Then we have the following lemma.

Lemma 12. The growth rates of each country's own capital will depend on the relative volume of domestic consumption and export. Precisely, let the growth rates of capital owned by Country 1 and Country 2 is g_1 and g_2, respectively, we have $g_1 > g_2$, if the value of Country 1's total consumption minus its export is less than, or not too larger than the corresponding value for Country 2; and vice versa.

Remarks: Here, we can see that the results for the different situation of both open-loop Nash equilibrium and closed-loop Nash equilibrium are very similar. All of them tell us the strictly different growth rates of consumption and conditional different growth rates of capital owned. Even the difference of growth rates of capital owned by an individual country is conditional, we still can find that the equivalence of both rates is very difficult, which only occurs for a special condition. The results above show the difficulty of convergence of growth rates, whatever in form of growth rates of consumption and/or growth rates of capital.

Now let us check such conclusions further for the released assumption of endogenously determined rates of time preferences. From Section 5.1, we have the following results first.

Lemma 13. The growth rate of consumption for each country is different. Moreover, at the steady state ($\dot{P}_x = 0$), the growth rate of consumption for the world is different, and depends on the relative greatness of the growth rates of consumption for each country.

That is, $\frac{\dot{C}_u}{C_u} = \frac{\dot{C}_c}{C_c} = \frac{\dot{C}}{C}, \; if \frac{\dot{C}_u}{C_u} = \frac{\dot{C}_c}{C_c}; \quad \frac{\dot{C}_1}{C_1} < \frac{\dot{C}}{C} < \frac{\dot{C}_2}{C_2}, \; if \frac{\dot{C}_u}{C_u} < \frac{\dot{C}_c}{C_c};$ and

$$\frac{\dot{C}_2}{C_2} < \frac{\dot{C}}{C} < \frac{\dot{C}_1}{C_1}, \; if \frac{\dot{C}_c}{C_c} < \frac{\dot{C}_u}{C_u}.$$

Lemma 14. Assuming that both countries have the same utility function form, then the growth rate of capital owned by the developing country (country 2) is greater than that of the world , and the latter is greater than that owned by the developed country (country 1); when the growth rate of capital used by each country is same as that of the world.

From Section 5.2, we can find under situation of closed-loop Nash equilibrium with endogenous rates of time preferences, we obtain the following results.

Lemma 15. The growth rate of consumption for each country is different, except that there exists the exactly same utility function with same parameters for both countries' consumers, and the same shadow price of different capital owned, whatever there are at steady state or not.

Lemma 16. Assuming both countries have the same utility function form, then the growth rate of capital owned by the developing country (country 2) is greater than that of the world , and the latter is greater than that owned by the developed country (country 1); when the growth rate of capital used by each country is same as that of the world.

7. Conclusions and Further Research Suggestions

We use differentiated game to explore the possible existence of the “unbalanced growth” with dynamic Nash equilibria in both one-way and two-way international capital flow. We find the wide existence of the “unbalanced growth rates (paths)” between the consumption and the capital owned by each country over time dynamically, regardless of the constant or endogenized rates of time preference and richness of countries. Moreover, we find with free flows of goods and capital internationally, the developed country obtains the significant benefit in the form of the higher growth rate of capital owned and low consumption growth rate, comparing with the developing country in the two-country world. It not only explains the “International consumption correlation puzzle” in general sense, but also explains the incentive to invest to the developing country from the developed country. The developing country, however, also benefits in such open economy since it will brings the higher growth rate of economy.

While the detailed results tell us more, our main results seem tell us the divergence of growth levels between the developing country and the developed country, although the growth rate of consumption could be adjusted to be different if the proportion of consumption between domestic and foreign goods is different.

In the further research, we could focus on the following two directions: (1) to re-test the “Kaldor’s facts” using more complete data, and (2) to introduce the endogenous growth model on such “unbalanced growth”.